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EQUILIZATION OF THE THERMISTOR RESPONSE

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of the Thermistor Response

Dr. B. Harris

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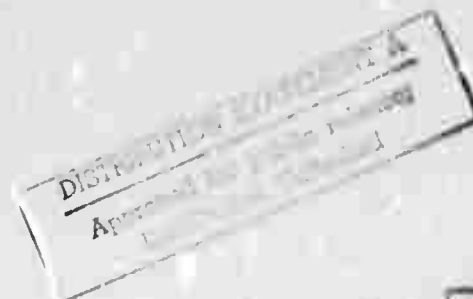Washington, D.C. 20301

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13. ABSTRACT

In the original setup for the NURDC Tower Experiment, the thermistor probes used to monitor the thermocline had time constants of the order of two minutes. Resolution of the data to within 0.5 minute was desired. Several ways of post facto equalizing the thermistor data are presented with the attendant advantages and disadvantages of each.

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Abstract

In the original setup for the NURDC Tower Experiment, the
thermistor probes used to monitor the thermocline had time constants
of the order of two minutes. Resolution of the data to within 0.5 minute
was desired. Several ways of post facto equalizing the thermistor data
are presented with the attendant advantages and disadvantages of each.

1.0 Introduction

In the NURDC Tower Experiment, thermistor probes were used to monitor the behavior of the thermocline. The time constants of the probes were to be about 0.5 minutes, but an extra layer of insulation resulted in time constants of the order of 2 minutes.

There are several ways in which the response of the thermistor probe can be post facto equalized so that the data will appear as if the correct thermistor response were present. The penalty for post facto equalization will be negligible of only phase equalization (time delay) is used, whereas there will be an increase (about 5 dB) in noise level if amplitude equalization is used as well.

The crudest form of phase equalization is simply a time shift. This may be sufficient. A more sophisticated approach is to equalize the phase response over the frequencies of interest. This can be done in either the time domain as a running weighted sum or in the frequency domain using the Fast Fourier Transform. The latter is preferred in that the impulsive response need not be calculated.

Another advantage of using the transform domain is that the amplitude can be equalized as well. This means that not only can the transients be properly aligned but that they can be sharpened as well so that the events can be more finely defined as to the time of occurrence.

2.0 Determination of the Thermistor Response

In order to equalize the response of the thermistor it is necessary to determine the response either by a set of measurements or by a theoretical approach. The actual response will, of course, be quite complicated and reasonable approximations must be used to obtain a feasible model.

2.1 Theoretical Response

As a basis for modeling the probe, the theoretical response will be obtained by considering the thermistor as a sphere of insulation material with the semiconductor being a small dot at the center. The temperature distribution within the probe, neglecting specific heat effects, is then governed by the classical heat equation for spherical symmetry

$$\frac{\partial (r\mu)}{\partial t} = a^2 \frac{\partial^2 (r\mu)}{\partial r^2}$$

in which r is the radial distance

μ is the temperature.

The initial conditions are

$$\mu = f(r) \quad @ \ t = 0$$

and the boundary conditions are

$$\mu = T_1 \quad @ \ r = R$$

in which R is the radius of the sphere.

The general solution

$$
r\mu = \frac{2}{R} \sum_{m=1}^{\infty} \left(e^{-\frac{m^2 a^2 \pi^2}{R^2}t} \sin\frac{m\pi r}{R} \int_0^R \lambda f(\lambda) \sin\frac{m\pi\lambda}{R}\, d\lambda \right)
$$

$$
+ T_1 \left[r + \frac{2R}{\pi} \sum_{m=1}^{\infty} \frac{(-1)^m}{m} e^{-\frac{m^2 a^2 \pi^2}{R^2}t} \sin\frac{m\pi r}{R} \right]
$$

for $f(r) = T_0$ reduces to

$$
\mu = T_1 + \frac{2R}{\pi r}(T_0 - T_1)\left[e^{-\frac{a^2\pi^2}{R^2}t} \sin\frac{\pi r}{R} \right.
$$
$$
\left. - \frac{1}{2} e^{-\frac{4a^2\pi^2}{R^2}t} \sin\frac{2\pi r}{R} + \frac{1}{3} e^{-\frac{9a^2\pi^2}{R^2}t} \sin\frac{3\pi r}{R} + \ldots \right]
$$

When t is small, the infinite series becomes

$$
\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} \sin\frac{k\pi r}{R} = \frac{\pi r}{2R}
$$

and $\mu = T_1 + \frac{2R}{\pi r}(T_0 - T_1)\frac{\pi r}{2R} = T_0$ as expected.

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When the thermistor bead is small such that $a^2t/R^2 >> 1$, the first exponential term quickly dominates the remaining terms of the series and

$$\mu = T_1 + \frac{2R}{\pi r}(T_0 - T_1) e^{-\frac{a^2\pi^2 t}{R^2}} \sin \frac{\pi r}{R}$$

which for $r \to 0$ becomes

$$\mu = T_1 + 2(T_0 - T_1) e^{-\frac{a^2\pi^2 t}{R^2}}$$

showing that the response, except for very small t, is governed by a single time constant.

To obtain a simple single time constant fit, the following modification will be made

$$\mu = T_1 + (T_0 - T_1) e^{-\frac{a^2\pi^2 t}{R^2} + \ln 2}$$

$$= T_1 + (T_0 - T_1) e^{-\frac{a^2\pi^2}{R^2}\left[t - \frac{R^2}{a^2\pi^2}\ln 2\right]}$$

Thus the response can be approximated by

$$\mu = T_1 + (T_0 - T_1) e^{-\frac{a^2\pi^2}{R^2}t'}$$

in which the new time axis, t', differs from the old time axis, t, by an amount

$$R^2 \ln 2 / a^2\pi^2$$

The corresponding thermistor response in the transform domain is

$$\frac{1}{(1+S\tau_o)}$$

plus a delay of $\tau_o \ln 2$. Formally, the transfer function is

$$\frac{e^{-S\tau_o \ln 2}}{(1+S\tau_o)}$$

An idea of the accuracy of the approximation can be obtained by considering the 50 percent point in the step transient response. The next term in the series is then $1/2\,(0.5)^4$ or 0.03. The error is thus about 6 percent.

It remains to adjust the model somewhat to account for the actual ratio of thermistor to insulation material. This corresponds to measuring temperature not at $r = 0$ but at some non zero radial location. The effect is to modify the delay by

$$\tau_o \ln \sin \frac{\pi r}{R}$$

Since this factor is not known and in actual fact the net delay could be negligible, the response will be taken as

$$\frac{e^{-S\tau_1}}{1+S\tau_o}$$

with the delay, τ_1 , handled separately should it not be negligible.

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2.2 Experimental Determination

The step response of the thermistor probe can be measured by maintaining the probe in air at a temperature T_0 until equilibrium is reached and then quickly plunging the probe into water at a temperature T_1. The response

$$\mu = T_1 + (T_0 - T_1)\, e^{-\frac{(t - \tau_1)}{\tau_0}}$$

can be fitted to the curve of temperature vs time by plotting ($\mu - T_1$) vs t on semi log paper and fitting a straight line. The delay τ_1 may prove to be negligible.

3.0 Data Processing Model for the Thermistor

Several models, of varying sophistication, can be used for equalizing the thermistor. The simplest is a delay, next a phase equalization, and lastly both an amplitude and phase equalization.

3.1 Delay Model

The thermistor response can be simply approximated by a single delay. Mathematically, for S (frequency) small the transfer function is being approximated by

$$\frac{e^{-S\tau_1}}{(1 + S\tau_0)} \approx \frac{e^{-S\tau_1}}{e^{S\tau_0}} \approx e^{-S(\tau_1 + \tau_0)}$$

This is quite valid if most of the energy or power content of the signal is below the 3 dB point of the response since the phase error at this frequency is only $\pi/4$ or i.e., the cosine of the error angle is 0.707.

3.2 Phase Equalization

Equalization of the phase response can be accomplished in either the time domain or the frequency domain. The frequency domain is conceptually easier because the impulsive responses of the equalization circuits do not have to be evaluated.

The correct phase equalization is a delay τ_1, if needed, plus the angle

$$\theta(\omega) = \tan^{-1}\omega\tau_0$$

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This can be done by using the FFT to obtain the Fourier Transform, applying the delay and angle equalization, and then using the Inverse FFT. There will be a truncation effect which will destroy the usefulness of the sample points near the ends of each batch of data. This truncation effect will propagate into the data samples about a time constant or a minute. To avoid this, the end points should be discarded and the FFT time-interleaved to cover this gap.

3.3 Amplitude and Phase Equalization

There is very little additional effort required to equalize both amplitude and phase vs phase alone. The penalty for post facto equalization of amplitude is an increase in the noise level. Indeed, this is what prevents complete equalization of the amplitude to a flat response.

At this point there are options. One can equalize the 2 minute response to look like a 0.5 minute response or one can, within limits determined by the deterioration of the noise level, equalize to other responses or even other bandshapes than the single time constant of a physical thermistor.

These options bring up the subject of what responses have good transients. For example, if one were to equalize to a rectangular low-pass transfer function with linear phase, the Gibbs phenomena would show a good rise time but an 8 percent overshoot. Other bandshapes would have poorer rise time but less overshoot. The single-time constant response of a physical thermistor would have a fair rise time with no overshoot.

In the interests of credibility in the resulting data, only the case of equalizing to a 0.5 minute response will be considered here. The procedure is

1. Sample as long a duration as the computer can handle.

2. Take the FFT.

3. Apply the transfer function

$$\frac{1 + j\, \omega\, \tau_0}{1 + j\, \omega\, \tau_0'}$$

to each frequency of the transformed data

τ_0 = two minutes = 120 seconds
τ_0' = half minute = 30 seconds

4. Take the Inverse FFT

5. Discard the two minutes of data at each end of the data block.

6. Repeat with the new data block overlapping the gaps.

7. Add a time shift, if required.

The sampling rate should be based on the wider bandwidth and signal content to avoid aliasing. As an example, suppose about one percent aliasing is allowed based only on bandwidth. The corresponding frequency is given by

$$\omega\, \tau_0' \approx 100$$

$$f \approx \frac{100}{2\pi(30)} \approx 0.5 \; Hz$$

The sampling rate is twice this or about 1.0 Hz or every second.

With a 4096 point FFT, about 4096 seconds of data are processed each time, i.e., 68 minutes. After the transfer function equalization is applied and the Inverse FFT taken, the end points must be discarded. The extent of the end points is determined by the longer time constants — about 2 minutes at each end. Removing these points still leaves an hour of data.

Processing time will mainly depend on the I/O as well as the time to take the FFT twice.

The noise level associated with this amplitude equalization changes by the ratio

$$\frac{\int_0^\infty \dfrac{df}{1 + (\omega \tau_0')^2}}{\int_0^\infty \dfrac{df}{1 + (\omega \tau_0)^2}}$$

With
$$x = \omega \tau_0'$$
$$y = \omega \tau_0$$
the ratio becomes

$$\frac{\tau_0}{\tau_0'} \cdot \frac{\int_0^\infty \dfrac{dx/y}{1 + x^2}}{\int_0^\infty \dfrac{dy}{1 + y^2}} = \frac{\tau_0}{\tau_0'}$$

This is a ratio of 120/30 or 4:1 in power — about 6 dB. Allowing that some extra signal power is present in the wider bandwidth ($\approx$ 1 dB), the noise level will probably deteriorate about 5 dB.